Unsteady free convective flow of water at 4 degree C and heat transfer through porous medium bounded by a isothermal porous vertical surface in the presence of variable suction and heat generation absorption.

Rachna Khandelwal
khandelwalrachna@ymail.com

Abstract

Unsteady free convective flow of water at 4 degree C through porous medium bounded by porous vertical isothermal surface with variable suction and heat generation absorption is investigated . The governing equations of motion and energy are transformed into ordinary differential equations and solved numerically using Runge-Kutta forth order method along with shooting technique. The effects of physical parameters on fluid velocity and fluid temperature are discussed and shown through graphs. The skin-friction coefficient and Nusselt number at the surface are derived discussed numerically and their numerical values presented through tables.

Keywords : Unsteady, variable suction, porous medium, heat generation, absorption, skin-friction, Nusselt number.

1.Introduction

Flow and heat transfer of an incompressible viscous fluid over a vertical porous plate appeared in several technological processes of industries such as nuclear science, fire engineering, combustion modeling, geophysical etc.

Ostrich (1952) obtained similarity solution of free convection flow along vertical plate . Soundalgekar (1971) and Soundalgekar and Wavre (1977) investigated unsteady free convection flow along vertical porous plate with different boundary conditions and viscous dissipation effect. Rapits and Tzivanidis (1981) studied mass transfer effects on heat transfer along an accelerated vertical plate. Hossain and Begum (1984) observed the effect of mass transfer and free convection past a vertical porous plate. Sharma (1992) investigate free Convection effects on the flow past a porous medium bounded by a vertical infinite surface with constant suction and constant heat flux-II. Crepeau and Clarksean (1997) obtained similarity solution of natural convection flow with internal heat generation, Sattar et al. (2000) analyzed analytical and numerical solutions for free convection flow along a porous plate with variable section in porous medium. Ferdows et al. (2004) obtained free convection flow with variable suction in presence of thermal radiation. Sharma and Mishra (2005) observed unsteady flow and heat transfer along a porous vertical surface bounded by porous medium. Sharma and Gupta (2006) analyzed unsteady flow and heat transfer along a hot vertical porous plate in the presence of periodic suction and heat source. Sharma and Singh (2008) studied unsteady MHD Free Convective flow and heat transfer along a vertical porous plate with variable section and internal heat generation. Sharma and Singh (2008) investigated free convection flow and heat transfer through a viscous incompressible electrically conducting fluid along an isothermal vertical porous non-conducting plate with time dependent suction and exponentially decaying heat generation in the presence of transverse magnetic field.

Aim of the paper is to investigate unsteady free convection flow of water at 4 degree through porous medium bounded by porous vertical isothermal surface in the presence of variable suction and heat generation/absorption.

2.Formulation of the Problem

Consider unsteady two dimensional flow of water at 4 degree C through porous medium bounded by a vertical non conducting surface kept at constant temperature T_w in the presence of variable suction and volumetric rate of heat generation/absorption. The x-axis is taken and y-axis is normal to the surface. Incorporating the Boussinesq approximation within the boundary layer [Schlichting and Gersten(1999),Bansal(2000)], the governing equations of continuity, momentum and energy respectively are given by

$$\frac{\partial v}{\partial y} = 0, \Rightarrow v \text{ is independent of y } \Longrightarrow = \text{v(t)}, \qquad \ldots(1)$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta((T - T_\infty)^2 - \frac{\nu}{K^*}u, \qquad \ldots(2)$$

$$\rho Cp\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty), \qquad \ldots(3)$$

where (u, v) are the velocity components in (x, y) directions, respectively. ν is the kinematic viscosity of the fluid, t is time , ρ is the density, g is acceleration due to gravity of Earth, K^* is the permeability, C_p is the specific heat at constant pressure, κ is the thermal conductivity and T is temperature of fluid in the boundary layer.

The boundary conditions are :

$$y = 0 : u = 0, v = v(t), T = T_w;$$

$$y \to \infty : u \to 0, T \to T_\infty; \qquad \ldots (4)$$

where T_∞ is fluid temperature for away from the plate.

3.Method of Solution

The momentum and energy equations can be transformed into the corresponding ordinary differential equations by introducing the following similarity variables and non-dimensional parameters:

$$h = h(t), v = v(t) = -V_0 \frac{\nu}{h(t)}, \eta = \frac{y}{h}, u = Uf(\eta), \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$S = \frac{h^2 \mu C_p Q}{\kappa (T_w - T_\infty)}, Gr = \frac{g \beta h^2 (T_w - T_\infty)}{\nu U}, K = \frac{K^*}{h^2} \text{ and } \Pr = \frac{\mu C_p}{\kappa}. \quad \dots(5)$$

Into the equations (1) to (3), we get

$$f'' + (\eta + V_0) f' + Gr\theta^2 - \frac{1}{K} f = 0 \quad \dots(6)$$

$$\theta'' + \Pr(\eta + V_0)\theta' + S\theta = 0 \quad \dots(7)$$

Where V_0 is suction parameter, η is similarity variable, U is uniform characteristic velocity, f is non-dimensional stream function, θ is the non-dimensional temperature, S is the volumetric rate of heat generation absorption parameter , Gr is the Grashsof number, K the permeability parameter and Pr the Prandtl number.

The boundary conditions are reduced to

$$f(0) = 0, \theta(0) = 1, f(\infty) = 0 \text{ and } \theta(\infty) = 0 \quad \dots(8)$$

The governing equations (6) and (7) are non-linear second order coupled differential equations and solved under the boundary conditions (8) using Runge Kutta fourth order technique with shooting technique.

First, the equation (7) is solved by transforming it into system of first order ordinary differential equations as given below :

$$\frac{d\theta}{dn} = f_1(\eta, \theta, w) = w, \theta(0) = 1, \quad \dots(9)$$

$$\frac{dw}{dn} = f_2(\eta, \theta, w) = -\Pr(\eta + V_0)w - S\theta, w(0) = ? \quad \dots(10)$$

and then the value of w(0), which is required to measure heat transfer at plate, is obtained using shooting technique. Once the value of w(0) is obtained,then the equations (9) and (10) become initial valued and can be solved using Runge-Kutta fourth order technique [Conte and Boor (1981),Jain, Iyenger and Jain(1985)]. After finding the solution of equation (7), the solution of equation (6) is obtained by the similar process.

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4.Skin-friction Coefficient

Skin-friction coefficient at the surface is given by

$$C_f = 2(\mathrm{Re})^{-1} f^{'}(0), \quad \ldots(11)$$

where $\mathrm{Re} = \frac{Uh}{\nu}$ is the Reynolds number.

5.Nusselt Number

The rate of heat transfer in terms of the Nusselt number at the plate is given by

$$Nu = -\theta^{'}(0). \quad \ldots(12)$$

6.Results and Discussion

Unsteady free convective flow of water at 4 degree and heat transfer through porous medium bounded by an infinite vertical porous surface with variable suction and internal heat generation / absorption is analyzed and solved numerically using Runge-kutta forth order method .The effect of the flow parameters on the velocity and temperature distributions are presented through figures. Numerical values skin-friction coefficient and Nusselt number are derived at the surface for different values of physical parameters and presented through tables 1 and 2 ,respectively.

It is observed from Figure 1 that fluid velocity decreases with the increase in the permeability parameter K and suction velocity V_0 ,while it increases with the increase in Grashof number Gr or in presence of heat source.

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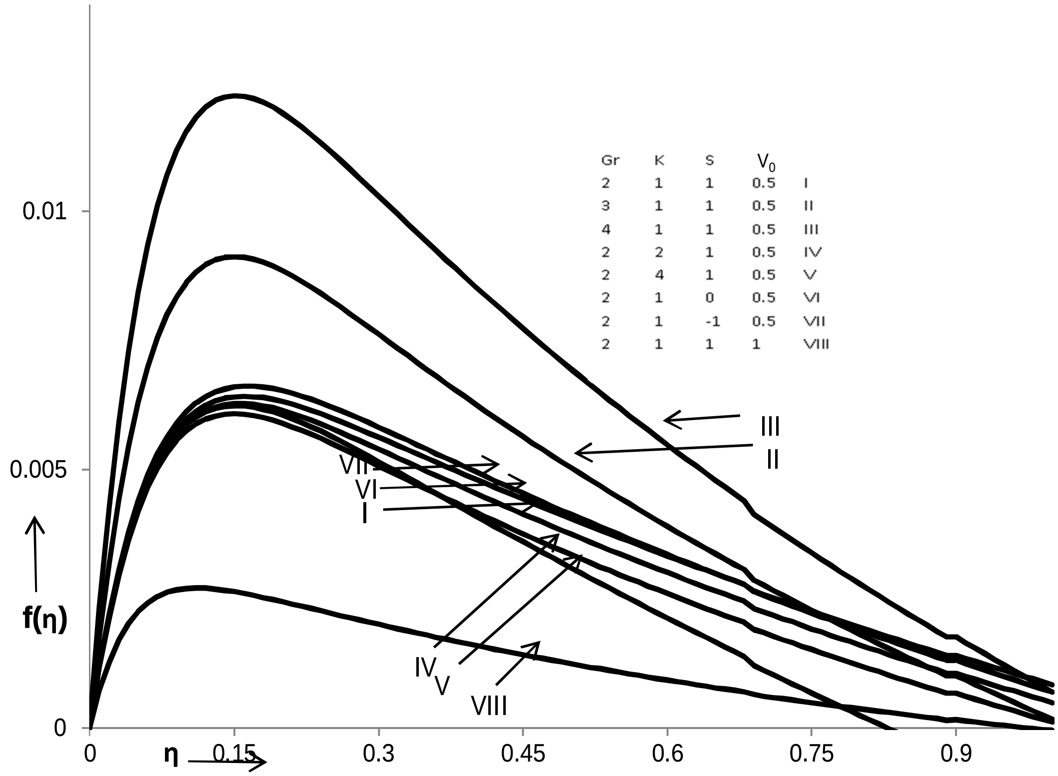

Fig.1: Velocity distribution versus η when Pr=11.4.

Figure 2 depicts that fluid temperature increases in case of heat source, while it decreases in case of sink

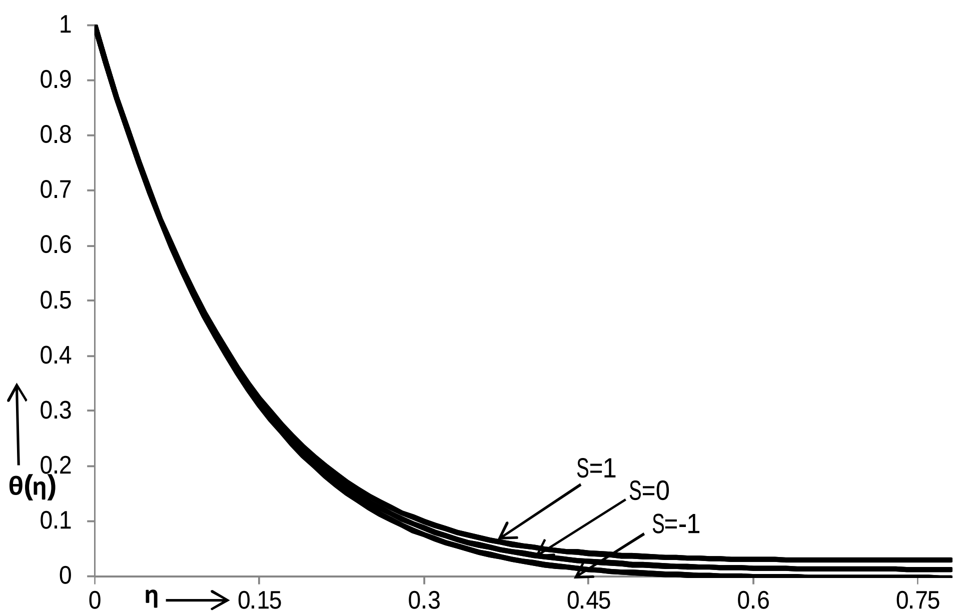

Fig.2: Temperature distribution versus η when Pr=11.4,V_0=0.5 .

It is seen from Table 1 that the numerical values of f '(0) for skin-friction increases with the increase in Grashof number Gr, Permeability parameter K, in presence of heat source, while it decreases with the increase in cross flow velocity when Prandtl number is fixed.

Table 1: Values of f'(0) for various values of Gr, K, S and V_0 when Pr=11.4.

Gr	K	S	V0	f'(0)
2	1	1	0.5	0.116247
3	1	1	0.5	0.174362
4	1	1	0.5	0.242959
2	2	1	0.5	0.117738
2	4	1	0.5	0.118735
2	1	0	0.5	0.117739
2	1	-1	0.5	0.120232
2	1	1	1	0.0811786
2	1	0	1	0.0812071
2	1	-1	1	0.0812072

Table 2 shows that the numerical values of (-θ'(0)) for Nusselt number increases with increase in cross- flow velocity.

Table 2: Values of (–θ' (0)) for various values of S and V_0 when Pr=11.4.

S	V_0	-θ'(0)
1	0.5	7.00225
0	0.5	7.00240
-1	0.5	7.00227
1	1	12.0228
0	1	12.0232
-1	1	12.0225

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